

Planned Contrasts

Following a significant one-way analysis of variance (ANOVA), the researcher may be interested in following up the analysis with some specific comparisons. In the case of the *planned contrast* or *planned comparison*, only a few predicted or *a priori* hypotheses are of interest, and familywise error is not likely to be a serious concern. Post hoc tests that adjust for familywise error typically follow a significant one-way ANOVA when many or all possible comparisons are of interest. Philosophically, the distinction between an *a priori* and *post hoc* test has to do with whether or not the group means compared were predicted to be different in advance or are decided after looking at the results. Statistically, the distinction also concerns whether there are a few or many contrasts conducted. Statisticians will cite either the philosophical or the statistical reason for deciding between the two approaches. We know that with many contrasts, familywise error becomes a problem, but if not very many contrasts are performed (e.g., 2, 3, 4?) than it is likely to be safe to use a planned contrast approach.

Planned contrasts typically involve the comparison of just two means. More complicated tests can be conducted (e.g., in a three group design, the average of two groups might be compared to the third group), but I will not get into demonstrating more complicated comparisons in this handout (see Keppel & Wickens, 2004, for more detail). The approach is to develop a set of weights that eliminate any group means that are not involved in the comparison by giving them a zero weight and to specify the group means to be compared by giving them opposite values, usually -1 and +1. Thus, the first step is to obtain the weighted sum, ψ , that gives the appropriate difference between the two means that one wishes to compare.

$$\psi = \sum w_j \bar{Y}_j$$

In the formula, \bar{Y}_j represents the group mean for each cell and w_i represents the contrast weights or “coefficients.” In a three-group design, a comparison between the second and third group means uses the weights of 0, -1, and +1. The first mean drops out because it is multiplied by 0. The next step is to compute the sum of squares.¹

$$SS_{\psi} = \frac{n\psi^2}{\sum w_j^2}$$

The mean-square, MS_{ψ} , is then obtained by dividing the sum of squares by the degrees of freedom. Planned contrasts of two groups (or the average of two groups compared to a third group) always have $df = 1$, so the sum of squares is equal to the mean square. The F-test is the ratio of the sum of squares contrast to the means squares from the one-way ANOVA (i.e., the error term from the full design), $MS_{s/A}$. The error term and sample size are based on all of the cases from the one-way ANOVA rather than just the two groups compared as with the t-test.

$$F = \frac{SS_{\psi}}{MS_{s/A}}$$

¹ The text book for this course (Myers & Well, 2003) refers to planned contrasts as t-tests. I've presented the F-test version here for two reasons: (a) to avoid confusion with the standard t-test that uses only the cases from the two groups that are compared; (b) because most textbooks and software packages use an F-test for the planned contrast, and (c) most articles will report an F-value. The two tests are statistically equivalent because $t_{\psi}^2 = F_{\psi}$.

I will use the teacher satisfaction example from the one-way ANOVA handout to illustrate the computation of the planned contrast: public (6) , charter (9), and private (6). The omnibus ANOVA was significant, and it might be desirable to follow the test with a comparison of the means for public and charter schools for theoretical or policy reasons. Thus, the comparison involves the first two groups, and the contrast weights should be -1, +1, and 0.

$$\begin{aligned}
 SS_{\psi} &= \frac{n\psi^2}{\sum w_j^2} & MS_{\psi} &= \frac{SS_{\psi}}{df_{\psi}} = \frac{22.5}{1} = 22.5 \\
 &= \frac{5(3)^2}{(-1)^2 + (1)^2 + (0)^2} & F &= \frac{SS_{\psi}}{MS_{s/A}} = \frac{22.5}{1.833} = 12.27 \\
 &= \frac{45}{2} = 22.5 & \sqrt{F_{\psi}} &= t_{\psi} \\
 & & \sqrt{12.27} &= 3.503
 \end{aligned}$$

The critical value with $df_{\psi} = 1$ and $df_{s/A} = 12$, is 4.75 for $\alpha = .05$. The calculated F-value of 12.27 is greater than the critical value, so the test is significantly different. The teachers satisfaction level differs in the public and charter schools.

To obtain this contrast in SPSS, a contrast subcommand can be added to the one-way ANOVA using the following syntax and this produces a t-test version of the planned comparison. The MANOVA command can also be used, but requires a -1 comparisons, where a is the number of levels.

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ONEWAY satisfaction BY school
  /CONTRAST= -1 1 0.
  
```

Oneway

ANOVA

satisfaction

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	45.938	2	22.969	5.742	.016
Within Groups	52.000	13	4.000		
Total	97.938	15			

Contrast Coefficients

Contrast	school		
	public	charter	private
1	-1	1	0

Contrast Tests

		Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
satisfaction	Assume equal variances	1	4.0000	1.21106	3.303	13	.006
	Does not assume equal	1	4.0000	1.31656	3.038	6.256	.022