

## Planned Contrasts

Following a significant one-way analysis of variance (ANOVA), the researcher may be interested in following up the analysis with some specific comparisons. In the case of the *planned contrast* or *planned comparison*, only a few predicted or *a priori* hypotheses are of interest, and familywise error is not likely to be a serious concern. Post hoc tests that adjust for familywise error typically follow a significant one-way ANOVA when many or all possible comparisons are of interest. Philosophically, the distinction between an *a priori* and *post hoc* test has to do with whether or not the group means compared were predicted to be different in advance or are decided after looking at the results. Statistically, the distinction also concerns whether there are a few or many contrasts conducted. Statisticians will cite either the philosophical or the statistical reason for deciding between the two approaches. We know that with many contrasts, familywise error becomes a problem, but if not very many contrasts are performed (e.g., 2, 3, 4?) than it is likely to be safe to use a planned contrast approach.

Planned contrasts typically involve the comparison of just two means. More complicated tests can be conducted (e.g., in a three group design, the average of two groups might be compared to the third group), but I will not get into demonstrating more complicated comparisons in this handout (see Keppel & Wickens, 2004, for more detail). The approach is to develop a set of weights that eliminate any group means that are not involved in the comparison by giving them a zero weight and to specify the group means to be compared by giving them opposite values, usually -1 and +1. Thus, the first step is to obtain the weighted sum,  $\psi$ , that gives the appropriate difference between the two means that one wishes to compare.

$$\psi = \sum w_j \bar{Y}_j$$

In the formula,  $\bar{Y}_j$  represents the group mean for each cell and  $w_i$  represents the contrast weights or “coefficients.” In a three-group design, a comparison between the second and third group means uses the weights of 0, -1, and +1. The first mean drops out because it is multiplied by 0. The next step is to compute the sum of squares.<sup>1</sup>

$$SS_{\psi} = \frac{n\psi^2}{\sum w_j^2}$$

The mean-square,  $MS_{\psi}$ , is then obtained by dividing the sum of squares by the degrees of freedom. Planned contrasts of two groups (or the average of two groups compared to a third group) always have  $df = 1$ , so the sum of squares is equal to the mean square. The F-test is the ratio of the sum of squares contrast to the means squares from the one-way ANOVA (i.e., the error term from the full design),  $MS_{S/A}$ . The error term and sample size are based on all of the cases from the one-way ANOVA rather than just the two groups compared as with the t-test.

$$F = \frac{SS_{\psi}}{MS_{S/A}}$$

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<sup>1</sup> The text book for this course (Myers, Well, & Lorch, 2010) refers to planned contrasts as t-tests. I’ve presented the F-test version here for three reasons: (a) to avoid confusion with the standard t-test that uses only the cases from the two groups that are compared; (b) because most textbooks and software packages use an F-test for the planned contrast, and (c) most articles will report an F-value. The two tests are statistically equivalent because  $t_{\psi}^2 = F_{\psi}$ .

*t*-tests in which only cases in two of the groups are compared are less preferable because they will generally have less statistical power. Statistical power is lower with the standard *t*-test than it is with the planned contrast for two reasons: a) the sample size is smaller with the *t*-test, because only the cases in the two groups are selected; and b) in the planned contrast the error term is smaller than it is with the standard *t*-test because it is based on all the cases from the ANOVA.

I will use the teacher satisfaction example from the one-way ANOVA handout to illustrate the computation of the planned contrast: public ( $M = 6$ ), charter ( $M = 9$ ), and private ( $M = 6$ ). The omnibus ANOVA was significant, and it might be desirable to follow the test with a comparison of the means for public and charter schools for theoretical or policy reasons. Thus, the comparison involves the first two groups, and the contrast weights should be -1, +1, and 0.

$$\begin{aligned}
 SS_{\psi} &= \frac{n\psi^2}{\sum w_j^2} & MS_{\psi} &= \frac{SS_{\psi}}{df_{\psi}} = \frac{22.5}{1} = 22.5 \\
 &= \frac{5(3)^2}{(-1)^2 + (1)^2 + (0)^2} & F &= \frac{SS_{\psi}}{MS_{s/A}} = \frac{22.5}{1.833} = 12.27 \\
 &= \frac{45}{2} = 22.5 & \sqrt{F_{\psi}} &= t_{\psi} \\
 & & \sqrt{12.27} &= 3.503
 \end{aligned}$$

The critical value with  $df_{\psi} = 1$  and  $df_{s/A} = 12$ , is 4.75 for  $\alpha = .05$ . The calculated *F*-value of 12.27 is greater than the critical value, so the test indicates a significant difference. The teachers' satisfaction level differs in the public and charter schools.

To obtain this contrast in SPSS, a contrast subcommand can be added to the one-way ANOVA using the following syntax and this produces a *t*-test version of the planned comparison. The MANOVA command can also be used, but requires a -1 comparisons, where *a* is the number of levels (see the simple effects analysis handout for details).

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ONEWAY satisfaction BY school
  /CONTRAST= -1 1 0.
  
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**Oneway**

**ANOVA**

satisfaction					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	30.000	2	15.000	8.182	.006
Within Groups	22.000	12	1.833		
Total	52.000	14			

**Contrast Tests**

	Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)	
satisfaction	Assume equal variances	1	3.000000000	.8563488386	3.503	12	.004
	Does not assume equal variances	1	3.000000000	...	3.000	5.882	.025