

## Missing Data and Missing Data Estimation

### Listwise Deletion

Until recently, listwise deletion has been the most common way of dealing with missing data in SEM. That is, complete data were required on all variables in the analysis—any cases with missing data on one or more of the variables was eliminated from the analysis. In the last few years, however, researchers have begun to use data estimation techniques when there are missing data among the variables in a structural model. And simulation studies convincingly show that when there are a lot of missing data, listwise deletion will have biased parameters and standard errors (see Enders, 2001, for an illustration).

### MAR and MCAR

A distinction of the type of missing data was made by Rubin (1976), who classified missing data as missing at random (MAR), missing completely at random (MCAR), or neither. Both MAR and MCAR require that the variable with missing data be unrelated to whether or not a person has missing data on that variable. For example, if those with lower incomes are more likely to have missing data on the income variable, the data cannot be MAR or MCAR. When data are not MAR or MCAR, missingness is sometimes said to be “nonignorable”. The difference between MAR and MCAR is whether or not other variables in the data set are associated with whether someone has missing data on a particular variable. For example, are older people more likely to refuse to respond to the income variable? The term MAR is confusing because data are not really missing at random—missingness seems to depend on some of the variables in the data set. In fact, missingness can even be related to the variable with missing data, as long as that relationship can be accounted for by other variables in the data set.

### Determining If Missing Data is MAR or MCAR

Practically speaking, it is quite difficult to determine if your data are MAR or MCAR. With a single variable that has missing data, it is not too difficult to determine if any of the other variables in the data set predict whether there are missing data on a particular variable. In practice, however, data will be missing on a number of variables, and so determining if other variables are related may be considerably complex. But the real importance is determining if missingness is associated with values of the variables that are missing data. Determining whether data are at least MAR may be quite difficult or impossible to do. In a recent discussion of missing data estimation, Schafer and Graham (2002) state: "When missingness is beyond the researcher's control, its distribution is unknown and MAR is only an assumption. In general, there is no way to test whether MAR holds in a data set, except by obtaining follow-up data from nonrespondents or by imposing an unverifiable model." (p. 152). With attrition over time, it may be possible to test whether missingness is associated with the value of the variable by examining whether the variable at Time 1 (i.e., with complete data) is associated with the missingness for that variable at Time 2. If data are missing on individual items from a scale, an approximate approach might be to attempt to show that missingness on particular items is unrelated to scale scores for that measure. In other circumstances, one may have to provide a theoretical argument that missingness is not associated with the variable or rely on information in the literature. There are many writings on missing data estimation, but few on how to go about determining if data are really MAR or MCAR.

### FIML

Probably the best missing data estimation approach for structural equation modeling is full information maximum likelihood (FIML), which has been shown to produce unbiased parameter estimates and standard errors under MAR and MCAR. FIML, sometimes called "raw maximum likelihood" or just "ML," is currently available in Amos, Mplus, Mx, and Lisrel. It requires that data be at least MAR (i.e., either MAR or MCAR are ok). FIML works by estimating a likelihood function for each individual based on the variables that are present so that all the available data are used. For example, there may be some variables with data for all 389 cases but some variables may have data for only 320 of the cases. The fitting function for FIML is computed by summing all the individual fit functions, and, thus, it is able to use all 389 cases. Rather than the traditional approach to calculating chi-square, FIML estimates two models, the  $H_0$  model and the  $H_1$  model. The  $H_0$  model is the "unrestricted" model, meaning that all variables are correlated. The  $H_1$  model is the specified model. The difference between the two log-likelihoods is used to derive the chi-square. This approach allows one to use all the available information in the variables.

Recent work illustrates that modeling potential causes or correlates of the variables with missing data has important advantages when data are only MAR, particularly when the association of those “auxiliary” variables

with the variable with missing data is high (e.g.,  $> .4$ ) and when the amount of missing data is large (e.g.,  $> 25\%$ ; Collins, Schafer, & Cam, 2001; Graham, 2003). Graham shows that two methods of modeling these auxiliary variables (either as dependent variables or correlated variables) are equally effective in reducing parameter biases, but that including auxiliary variables as correlates has a greater impact on reducing biases in model fit. Furthermore, some missing data experts suggest that using missing data estimation may be better than listwise deletion even if data are not MAR

## Other Missing Data Approaches

**Multigroup SEM Approach.** Another approach to missing data analysis uses a multigroup structural model approach, suggested by Muthen, Kaplan, and Hollis (1987). The same model is estimated in different groups. The groups are based on different patterns of missing data—one group for each pattern. A few hand calculations must be done. This is a fairly impractical approach if there are many patterns of missing data, but might be especially useful if data are missing by design. This approach has been superseded in some cases by a latent class approach to missing data (Muthen & Muthen, 2002).

**Pairwise Deletion.** Pairwise deletion is sometimes used to estimate models when there are missing data. With pairwise deletion, a covariance (or correlation) matrix is computed where each element is based on the full number of cases with complete data for each pair of variables. This approach may lead to nonpositive definite matrices and to standardized values over 1. There are other potential problems with the approach and I do not recommend it.

**Other imputation methods.** There are several other estimation approaches in which the data are imputed. That is, a full data set is created based on the imputation method that fills in data based on information from existing data. Older methods, such as mean imputation (the average scores is filled in), regression-based methods (a regression is used to predict a score), and resemblance-based “hot-deck imputation” (which imputes new values from similar cases) do not perform as well as other methods, and some may produce highly biased coefficients and/or standard errors (Gold & Bentler, 2000). Two newer methods, multiple imputation (see Graham & Hofer, 2000) and Expectation Maximization (EM; which is a maximum likelihood-based approach; see Enders & Peugh, 2004), seem to perform much better, but are likely to be less convenient because they must be done separately from the test of structural model.

## Comments

When there is a large amount of missing data, researchers are better off using a FIML approach to estimation. Given that it is fairly easy to implement in the packages where it is available, there is no reason *not* to do it. What is a large amount of missing data? The percentage of missing data is sometimes discussed based on the percentage missing for a certain variable. It makes more sense to me to examine the percentage of cases missing if listwise deletion were to be used. With this method, data sets (i.e., the set of variables in the model) in which more than roughly 20% of the cases are excluded by listwise deletion seem to lead to substantial bias in estimates (e.g., Arbuckle, 1996). With fewer than this much missing data, it may not be a major difference whether listwise deletion or FIML is used. Missing data estimation can be used when data are nonnormal as well using scaled chi-square and robust standard errors (Yuan & Bentler, 2000). In Mplus, *estimator=MLR* is used to obtain the robust estimates with missing data.

## References and Further Readings

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